Patterns Of Thinking Errors In Construction Mathematical Proof With Cognitive Map

Anton Prayitno, Febi Dwi Widayanti

Abstract: This study aims to capture students' thinking error in the construction of mathematical proof, therefore this research is classified as a mixed methods. Student errors in mathematical proof are a reflection of her thinking. If these errors are not resolved, it will have an impact on students' thinking when working on further mathematical proof. This research was conducted on mathematics education students by asking students to complete proof of rational numbers. The results showed that the proof of thinking error occurs when students provide proof by providing a number or an example of a certain number. Actually students are able to do the proof that is given at the beginning of completing, but the resulting proof of the answer changes by entering numbers into the proof. The mistakes made by students are not only limited to corrections, but must be followed up by strengthening concept understanding and mastery of techniques and mathematical proof strategies.

Index Terms: thinking error, mathematical proof, cognitive map, pattern of thinking error, rasional number, mixed methods.

1 INTRODUCTION

The importance of mathematical proof is explained by several experts, for example [1] and [2] state that students need to improve their ability to construct mathematical proofs. The ability to compile evidence is closely related to thinking. Furthermore, [3] and [4] state that mathematical proof is the essence of mathematical thinking. Therefore, evidentiary learning is the key to learning mathematics as a whole. According to [5], [6] developing a theory of three worlds which is used to explain student thinking in compiling evidence, namely (1) form, real interactions that develop based on their experiences, (2) symbolic, manipulation of symbols that function properly, and (3) formal, building a system based on formal proof. Based on the results of previous research [7], it was found that many students had difficulty in proving when learning functions and limits, including: (1) students could not use definitions to compile proof, (2) students did not know how to start proofing, and (3) students have too little understanding of the concept, resulting in difficulty in constructing to proof. The difficulty of students in compiling evidence is not sufficient in terms of the evidence produced. The thinking processes that occur when compiling evidence can provide better clues to identifying student difficulties. Several researchers have previously examined students' thinking processes in terms of mathematical proof. Among them are the results of the study [8] showing the difficulties faced by students when writing evidence due to a lack of understanding of mathematical concepts; The results of this study are still limited to tracing the difficulty of writing proof and not yet examining how to correct these errors. Students' difficulties in learning mathematics are often represented by errors in solving mathematical problems. Mistakes in thinking often appear in the form of mistakes they make [9],[10]. Students' mistakes in proof of mathematics are a reflection of their way of thinking. If the error is not resolved immediately, it will have an impact on students' thinking when working on advanced mathematical proof. Because of that, it requires structuring thinking on existing knowledge so that it can correct the mistakes it makes. Several researchers have previously examined the difficulties of students and students' thinking processes in constructing mathematical proof, such as the difficulties faced by students when writing proof due to a lack of understanding of mathematical concepts [8]. In addition, students fail to construct mathematical proofs because they cannot use the strategies they have [11]. Other patterns of errors made by students in constructing mathematical proof are: 1) proving statements by providing numbers or examples, 2) manipulating incorrect algebra, 3) verifying numerical proof after formal proof, and 4) inability to understand the definition of the statement [12]. To be able to trace students 'thinking error in constructing proofs, it requires knowledge of how to photograph students' thinking. In the results of his research [13] explained that to deeply study the uniqueness of the thinking process, cognitive map / cognitive style can be used. Cognitive map is a technique to represent how the subject thinks about a particular problem or situation, so that researchers can act for the next step [14]. Cognitive map is a person's perspective on the subject, which is described qualitatively by connected concepts to predict causal behavior. Therefore, students' thinking processes in solving math problems can be traced by using cognitive map. Cognitive map can be described as follows.

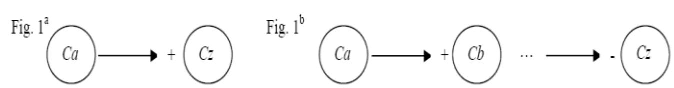

***Figure 1**. The Model of Cognitive Map [15]*

Some researchers traced students' thinking processes using a cognitive map. [15] explain that cognitive map can describe the causal relationship of various phenomena and concepts and can be modeled; [16] revealed that cognitive map can be used as a guide to the next step in order to obtain the next direction of thinking; [17] explains that the cognitive style or thinking style is used as a mediator for student work in solving geometry problems with Van Hielle's theory; [10] traces students' mistakes in constructing mathematical concepts with cognitive map; and cognitive map can be used to trace students' thinking processes in solving decision-making [18]. This article discusses the results of a study conducted on students working to prove mathematics. The study was conducted to see in more detail the types of errors of students, especially from the aspect of thinking when they constructed mathematical proofs. By knowing the types of mistakes committed, a scaffolding design or remedial scheme can be designed to be used to restructure students' thinking.

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Therefore, the purpose of this study is to examine the construction errors of mathematical proof using cognitive map.

2 Research Methods

The approach used in this research is a mixed method, which combines quantitative and qualitative approaches. The quantitative approach is carried out to process data related to how much students make mistakes in mathematical proof, while the qualitative approach is used to explore how students' mistakes occur in constructing mathematical proofs. This qualitative approach is also intended for how to restructure the thinking scheme so that it can be used to correct construction errors. Subjects were taken from the second batch of mathematics education students (semester 4). There are several reasons for choosing the subject, including: (1) students have learned the basic concepts of proof; (2) research does not interfere with lecture activities; and (3) students are still not burdened by thesis preparation. The main instrument is in the form of several mathematical statements that must be proven by students. Every proof must be followed by a reason. After completing the main instrument, students are asked to justify the results of the proof. Errors made by students while working on the instrument are grouped and patterns identified. The error pattern found is used to trace more deeply about the characteristics of students' errors in mathematical thinking. In-depth tracking was mainly carried out on several research subjects based on the error patterns made. The subject was given the main instrument as well as an interview for further investigation. Furthermore, the results of the interview were used to describe the students' cognitive map, including the identification of mistakes.

3 Result and Discussion

In this article, we will examine the mistakes of students thinking in proving rational numbers by providing certain numbers. Mistakes in thinking in this pattern occur when students provide proof by providing numbers or examples of certain numbers. Actually students are able to do the proof that is given at the beginning of completing, but the resulting proof of the answer changes by entering numbers into the proof. This, of course, shows that the proof is only valid for certain numbers. When answering questions of proof on research instruments that ask for proof of rational numbers. The response given by students is 66% proving it by providing certain numbers and numbers [12]. That is, in this proof, students choose a certain number to solve it. However, if it is identified from the beginning, the steps are correct in proving, most students assume that 5/6 or 1/3 are rational numbers. In this case, the rational numbers are only 5/6 or 1/3.

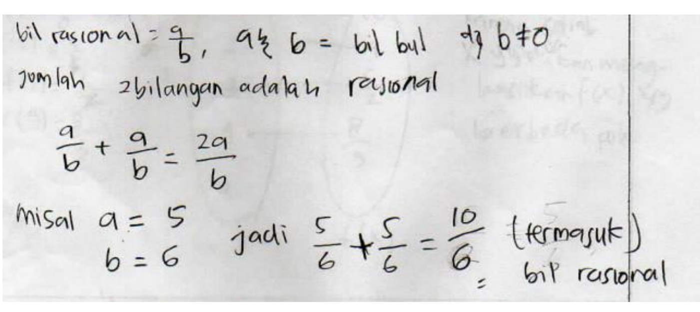

Figure 2. *The result of student work when contruction of mathematical proof*

In this case, students interpret rational numbers with numbers (5/6 or 1/3). When there is a statement about the sum of two rational numbers, students directly operate the two rational numbers. In this case, the student wrote 5/6 + 5/6 = 10/6. Likewise with other students, assuming that proving the sum of two rational numbers is rational can be proved by 1/3 + 2/5 = 11/15. In this case, students construct mathematical proofs using certain numbers. Student error in constructing a mathematical proof in the form of a specific number (ie, 5/6). Thus, students use this as an example in compiling mathematical evidence. If, the number is used as a process of proof, the proof only applies to the numbers 5/6. They do not understand that this evidence applies in general. Another mistake of students is also when they answer a / b + a / b = 2a / b is not a rational number. This is because the numbers 2a / b consist of natural numbers (i.e., 2) and rational numbers (a / b). Of course, they do not yet understand the definition of rational numbers (a / b; a and b is integers). This error also continues when students verify the proof with numbers (for example a = 5 and b = 6). Students think that a / b can be replaced by 5/6. "The five-sixths that the students thought were rational numbers defined as 5 and 6 integers. The next process, they continued by writing 5/6 + 5/6 = 10/6. Because 10 and 6 are integers and 6 are not the same. with zero, then 10/6 is considered a rational number. When viewed from the results of the work, this shows inconsistencies in the answer. In proof, students conclude that the sum of two rational numbers is not a rational number because it produces 2a / b, but when verified with the numbers 10/6 and it is concluded as a rational number. Students only understand that 1/2 or 5/10 are rational numbers, while 2 or 0 whose divisor is 1 is not a rational number. From the process of solving the problems described above, students experience the thought process of thinking errors in mathematical proof related to rational numbers. If the flow of the student's thought process is described through the stages (1) making the definition of rational numbers, (2) operating rational numbers by addition, (3) making rational numbers with certain numbers or numbers, (4) operating by adding up the two numbers and (5) identifies the result of operations with the general form of a rational number. The flow of thinking about students 'mistakes in proving students' rational numbers is depicted in Figure 2.

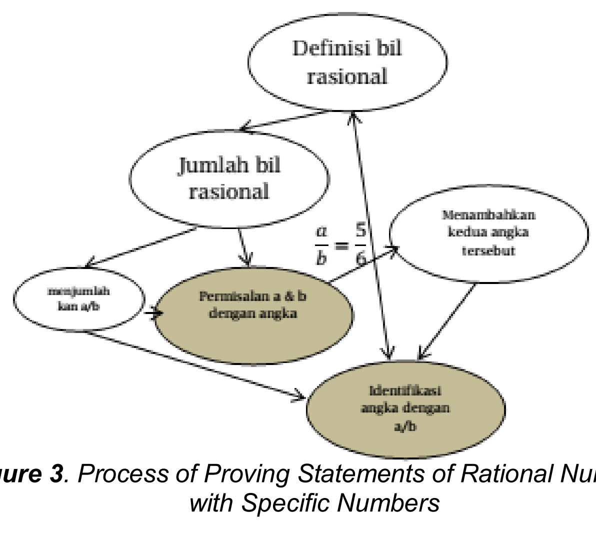

Figure 3. *Process of Proving Statements of Rational Numbers with Specific Numbers*

In this proof, students fail to understand the definition of rational numbers. This failure is also indicated by the results of the interview excerpt.

P: Is 2a / b a rational number?

S: No, because the rational number is a / b while 2 is a natural number. So 2a / b is not a rational number.

This is closely related to one of the conceptions of variables which are "something" that is "meaningful and generalizable" which involves numerical thinking process [19]. The facts found are also related to the statement [20] that when a variable is replaced by a certain number it shows the failure of the transition from arithmetic to algebra and the incorrect conception of the variable includes variables that simultaneously represent various numbers and the absence of positional value [21].

4 Conclusion

Student thinking errors in proving rational numbers by giving certain numbers are failure of students' thinking. This is certainly influenced by the inability of students to understand the concept of rational numbers and the concept of proof in general. The thinking errors made by students are not only limited to corrections, but must be followed up by strengthening concept understanding and mastery of techniques and mathematical proof strategies. Therefore, information about student errors should be an important input in the lecture process, so that lecturers can provide proper guidance to students who are experiencing difficulties, for example scaffolding.

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