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Concept Understanding Layers of Seventh Graders Based on Communication Ability in Solving Fraction Problems

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Abstract. This research aims at exploring layers of concept understanding manifested in junior high school students' communication skill when solving fraction problems using descriptive qualitative research design. The participants of this research are 22 seventh grader students. The instruments used include visual and verbal fraction problem sheets and unstructured interviews. The data collection procedure was started by providing the problem sheets to 22 participants. There are 4 indicators of communication skill in this research namely: (a) being able to make image sketch as a form of mathematical modeling which represents the fraction value clearly and completely, (b) identifying and explaining the fraction name from the image formed, (c) associating the data in order to obtain ideas for strategy composition in solving fraction problem systematically, and (d) providing explanation or statement verbally and in writing of each idea in solving fraction problems. Subjects with high communication skills have reached layer of understanding primitive knowing, image having, image making, property noticing, formalizing, observing, structuring and subject has passed the second "don't need" boundaries in solving fraction problems. Subject with moderate communication skills reached layer of understanding primitive knowing, image making, image having, property noticing, and observing. The subject of low communication skills reached layer of understanding primitive knowing, image making, property noticing, and formalizing. This research concludes that the students who have high communication skill are able to reach the deepest understanding layer such as primitive knowing and the outermost of understanding layer such as structuring, as well as pass the second "don't need" boundaries in solving mathematical problems.

INTRODUCTION

Pirie & Kieren (1) state that an understanding is a process of growth that is whole, layered but not linear, and endless. Understanding is also a dynamic and organized process. In developing their understanding, the students are on a particular layer and continue to progress in accordance with the level of growth in their understanding. One student with another student can be in a different understanding layer when learning the same material. Those understanding layers are primitive knowing, image making, image having, property noticing, formalizing, observing, structuring, and inventising. The growth in mathematical understanding is an acquired process rather than knowledge and according to the theory of Pirie-Kieren, it is a purposive model used to describe students' mathematical understanding as they engage in various levels (2), (3), (4). The accuracy in understanding the meaning of mathematical terms is important as not to cause misinterpretation and create ambiguity of an idea (5). According to Hidajat et al., (6), one indicator of students experiencing confusion in solving problems because they are not used to

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imagining and being creative on non-routine questions. A communication is said to be effective when the content of the message can be received and understood (7). Besides understanding the meaning, the students need to organize the ideas that they have so that they are capable of communicating clearly (5). It should be noted that for students, it is very important to plan what will be written and said before communicating it. It is intended so that the ideas conveyed are precise and accurate. On top of that, the recipient can understand the content of the idea. A student is considered to be ready to communicate if they are able to interpret and plan what will be explained. The students articulate thoughts and ideas clearly and effectively through speaking and writing (7). Writing helps to train the students to think about making decisions and practicing creativity whereas communicating the problems helps the students to deepen their understanding. In fact, learning content is enriched by critical thinking, collaboration, communication, and creativity. In addition, those things are parts of learning.

Baxter et al., (8) statement that communication skills are the most important component in learning mathematics. Mathematics is a social activity which requires active student interaction. One of them is by reading. The students can remember, understand, and analyze which involves psychomotoric activities (9). Based on NCTM (10) it is stated that the characteristics of mathematical communication include: the ability to read, write, listen, think, and communicate about problems in order to deepen the students' mathematical understanding. The students have the opportunity to present ideas and problem solving strategies clearly in communication (9). The other benefit gained by mathematical communication is that students are able to improve understanding, organize thoughts and connect mathematical ideas. It also helps the teachers know the concept and procedural errors made by the students (11). Considering the importance of mathematical communication in learning mathematics, the Government of Indonesia undertook several development actions related to students' mathematical communication in school. These actions are contained in Permendikbud No. 60 in 2014 about Curriculum 2013 which states that the fourth goal of learning mathematics is that students are able to communicate ideas, reasoning and be able to compile mathematical proof by using complete sentence symbol, table, diagram, or other media in order to clarify the situation and problem. The introduction that has been described previously shows that it is important to analyze the concept understanding and understanding in solving mathematical problem through the students' communication skill.

The understanding layer described in this research is based on the students' communication skill in solving mathematical problem. Students' written communication skill is analyzed using QCAI (*Quasar Cognitive Assessment Instrument).* QCAI is able to measure the three components specified, namely mathematical knowledge, problem solving strategies and communication (12). The researchers modified the analysis criteria of qualitative communication skill based on the appearing QCAI criteria. QCAI analysis includes four assessment criteria which have been adjusted to the needs of research namely 1) providing a complete and clear response, 2) making a complete and clear picture/mathematical modeling, 3) writing a problem solving strategy effectively and systematically, 4) providing string and logical supporting arguments.

Several previous studies on layers of understanding using Pirie - Kieren's theory have been conducted. Droujkova, et, al., (13) studied the conceptual framework for teachers and found collective understanding. Slaten (14) examined the effectiveness of learning geometry. Parameswaran (15) studied the tools used by mathematicians in developing understanding of definition. Sagala (16) studied the concept understanding layer profile of derivative function and the form of folding back of students of teacher candidate who have high mathematical skill based on gender. Utami & Rosyidi (17) examined the students' property noticing understanding layer profile on logarithmic material in terms of gender differences. Fauziyyah & Kriswandani (18) in their research described the concept understanding layer profile of the cone section and Rahayuningsih, et al., (19) examined the fraction concept understanding layer profile of Elementary School students. So further research is still needed to analyze the development and movement of the layers of student understanding from various points of view.

This research aims at exploring the concept understanding layer from seventh grader students according to their communication skill in solving fraction problems.

METHOD

The research design used to explore the concept understanding layers from seventh grader students based on their communication skill in solving fraction problems is the descriptive qualitative research design. The researcher made 2 rubrics in order to measure students' communication skill and explore the concept understanding layers of seventh grader students based on their communication skill in solving fraction problem. The assessment rubric used in this research is holistic assessment rubric. It is due to the fact that the holistic assessment rubric assesses the overall results of students' work based on quality, while also emphasizing the process of thinking and communication in mathematics

(20). The instrument used in this research includes the test questions and unstructured interviews. The test instruments are two fraction problems such as fraction problem related to visual problem and fraction problem related to verbal problem. The questions used as the test instrument in this research are shown in Fig. 1.

Selesaikan soal berikut dan berikan alasan yang tepat!

Pertanyaan	Jawaban	Alasan
1. Berapa nilai pecahan yang ditunjukkan pada gambar segitiga yang di arsir? 		
Pertanyaan	**Jawaban**	**Alasan**
2. Gambarlah persegi panjang yang menghasilkan nilai pecahan $\frac{3}{7}$?		

Solve the following problems and give the right reasons!

Question	Answers	Reasons
1. What is the fraction that shown in red triangles in all of triangles? 		
Question	**Answers**	**Reasons**
2. Draw a rectangle that shows $\frac{3}{7}$?		

FIGURE 1. Test instruments

The research subjects were selected based on students' communication skill that appeared. The result of students' answer was then analyzed using 4 criteria of QCAI in order to measure students' communication skill in solving fraction problem. The data collection procedure begins with giving test questions to 22 seventh grader students of SMP. The written results from the students' work were analyzed based on the QCAI criteria indicators are summarized in Table 1.

TABLE 1. Indicators of communication skill based on QCAI criteria

QCAI Criteria	Indicator
Making a complete and clear picture/mathematical modeling	Able to sketch pictures as a form of mathematical modeling which represents the value of fractions in a complete and clear way
Providing a complete and clear response	Able to identify and explain the name of the fraction from the image formed
Writing an effective and systematic problem-solving strategy	Able to connect data to get ideas to strategize in solving fraction problem systematically
Providing strong and logical supporting arguments	Able to give explanations or statements verbally and write down each idea in solving fraction

The results of the subjects' answer were then analyzed based on the Pirie - Kieren's model of understanding layer indicators which consist of eight understanding layers. Then triangulation method was conducted through interviews according to the answer result from the subjects in order to dig deeper information. The indicators or students' concept understanding layer in solving fraction problems from Pirie – Kieren's model is provided in Table 2.

TABLE 2. Indicators of students' understanding layer in solving fraction problems

Understanding Layer	Indicator
Primitive Knowing	Can understand the fraction concept in the form fraction value and those represented in the form of picture.
Image Making	Can provide a name or make fraction number symbol which is represented from a picture, and Can make pictures or represent fraction numbers into a picture.
Image Having	Have ideas to construct mental images in solving fraction problems.
Property Noticing	Can manipulate and combine the concept of fraction and division in the form of representations based on the nature of the operation numbers and the partial concept from the whole.
Formalising	Able to give meaning to each fraction as well as the one represented and are able to draw formal conclusions as a form of generalization from the fraction concept.
Observing	Can express problems related to the concept of fractions in various representations and use them in real life according to the actual information.
Structuring	Able to link relationships between theorems and are able to prove them based on logical reasons.
Inventising	Able to create new questions which can bring up new concepts based on the structure of previously possessed knowledge.

RESULTS AND DISCUSSION

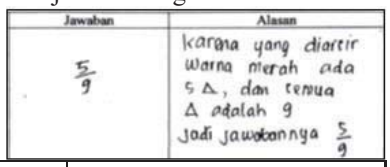
The criteria of Subject is summarized in the Table 3 below.

TABLE 3. The criteria of subject

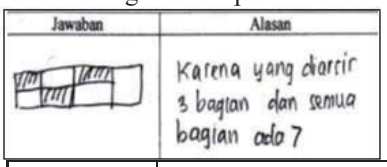
Subject	Indicator of Communication Skill
Subjects with high communication skills	4 indicators appear on the solution of both problem question type
Subjects with moderate communication skills	4 indicators appear only on the solution of 1 problem question type
Subjects with low communication skills	Fewer than 4 indicators appear in solving both types of problems

The Work Result of Subject with high communication skills in Solving Fraction Problem

The work result of subject with high communication skills in solving fraction problem is shown in Fig. 2.



Answers	Reasons
5/9	Because there are 5 Δ shaded in red, and all of Δ are 9 So the answer is 5/9



Answers	Reasons
	Because the shaded area is 3 and all parts are 7

FIGURE 2. The work result of subject with high communication skills

The result of the subject with high communication skills' answers shows can provide clear and logical reasons. The reasons written reflect that the subject understands the definition of a fraction which is represented in the form of a triangular image which indicates is in the primitive knowing understanding layer. This layer describes the process of initial efforts to understand a new concept by bringing previous knowledge to the next understanding layer which involves definitions (1), (15), (21). This subject also has the idea of constructing mental images in solving fraction problems which means that the subject also in the image having understanding layer (1). Fig. 2 (a) shows that subject can name the fraction represented in the triangular image and create a triangle symbol (Δ). It indicates that she is at the image making understanding layer. According to Martin, et al., (22) they state that in the image making understanding layer, students work by involving images/diagrams and through specific examples to develop their ideas. This subject can also be considered as in the property noticing understanding layer because the reasons given also indicate that the subject appears to be able to manipulate and combine the concepts of fraction and division in the form of representation based on the nature of number operation and the concept of parts from the whole. In line with

Sengul & Argat (23) statement, students in the property noticing layer can recognize the similarities and differences of the concepts they learn through the images of particular topic and develop them into a concept definition built from the relationships of these images.

P : Based on the answer you have written, what is the definition from $\frac{a}{b}$, if a, b are integer

S : a is a part from b part

P : Alright, if so from $a \neq 0$ and $b > a$, what can you conclude? If a, b are integer.

S : $\frac{a}{b} = a$ is a part from b part, $b \neq 0$ and $b > a$

P : Can you make one real example from the fraction that you defined?

S : Mm,, let's say that a cake will be given to 4 people, then each person will get $\frac{1}{4}$ part.

P : If so, what is the definition of fraction number $\frac{a}{b}$ other than a part from b part in your opinion?

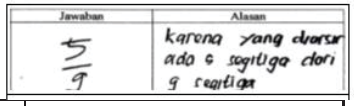
S : The fraction number is the number which is stated as $\frac{a}{b}$ with a and b are integer and $b \neq 0$ with a is called the numerator and b is called denominator.

The excerpt of interview shows that subject with high communication skills can give meaning to any fraction or the ones represented as well as draw formal conclusion as a form of generalization of fraction concept without having to show a specific image. It indicates that subject is in the formalizing understanding layer but does not require a description (property noticing). In the formalizing layer students are ready and able to clearly state as well as appreciate a definition or formal mathematical algorithm (1). According to the previous excerpt, the subject has passed the second "don't need" boundaries in solving fraction problems because this subject has formal mathematical ideas and does not require an image from the property noticing. Based on Pirie & Kieren (1), they state that when "don't need" boundaries occur, the students no longer need specific actions which have been taken inside the boundary level and they can work with a more general and abstract level of understanding beyond the boundaries. More specifically, according to Pirie - Kieren's theory, formalizing occurs just outside the second "don't need" boundaries (25).

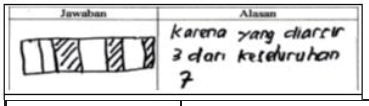
This subject can also express problems related to the fraction concept in various representations and use them in real life. Thus, she is also in the observing understanding layer. In this layer, the students can reflect and coordinate formal activities in the formalizing so that they are able to use it on the problems faced and express it as theorem (1). In the last part of the previous interview excerpt regarding the other meaning of fractions expressed as $\frac{a}{b}$ with a and b are integer and $b \neq 0$ with number a is called as numerator and number b is called denominator which means that subject is in the structuring understanding layer. It is because she is able to associate the relationships between theorems and is able to prove them based on logical reasons. The students think about formal observation as theory and put forward logical arguments in the form of evidence (1).

The Work Result of Subjects with Moderate Communication Skills in Solving Fraction Problems

The result of S2's answer in solving fraction problems can be seen in Fig. 3.



Answers	Reasons
$\frac{5}{9}$	Because there are 5 shaded triangles among 9 triangles



Answers	Reasons
	Because the shaded areas are 3 and the whole areas are 7

FIGURE 3. The work result of subject with moderate communication skills

The result of subject with moderate communication skills answer shows that the subject can provide clear and logical reasons for solving problems related to visual problems. However, this subject could provide reasons even though it was unclear and less logical in solving fraction problems related to the verbal problems. The reasons written reflect that the subject understands the definition of a fraction which is represented in the form of a triangular image which indicates that the subject in the primitive knowing understanding layer. This layer describes the process of initial attempts to understand a new concept by bringing previous knowledge to the next understanding layer which involves definition (1), (15), (21). The subject could give name $\frac{5}{9}$ to fractions which were represented by triangular

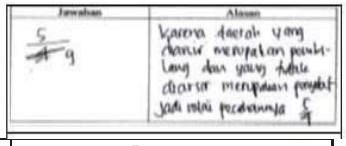
images and make images of fractions $\frac{3}{7}$. It indicates the subject at the image making understanding layer. Parameswaran (15) states that in this image making layer, the students can create an image from their previous knowledge. Meel, (26) emphasizes that in the image making layer, the students try to understand a topic, both mentally and physically, in order to be able to get an idea about the topic. As a result, the actions in this layer involve developing a relationship between images and symbols. This subject at the image having understanding layer because had the idea of constructing fraction $\frac{5}{9}$ in solving fraction problem related to visual problems. Besides, it can be said that the subject is consistent in comparing the two-digit values indicating the level of synthetic mental models (27). He had the idea of making a rectangle as a mental image in solving fraction problems related to verbal problems. In the image having layer, the topic image generated from the previous understanding layer is replaced by a mental image (26). The intention of this subject reasons to mention shaded fractions and their whole indicates that subject at the property noticing understanding layer. It is because the subject can manipulate and combine the concepts of fraction and division in the form of a representation based on the nature of number operation and the concept of part the whole. In the property noticing layer, the students can manipulate and combine aspects of a topic to form properties which are relevant and specific to the concept (1).

P : Let's make it like this, say for example an orange is peeled and it turns out there are 12 segments. Because it has to be given to 3 people, how many segments should each person receive so that it is equal?
S : Each person should get 4 segments mam.
P : What is the reason?
S : $\frac{12}{3}$ mam
P : The previous 4 segments are equal to how much part of the whole orange?
S : It means 4 out of 12 segments mam, that is $\frac{4}{12} = \frac{1}{3}$
P : If that so, the definition of fraction $\frac{a}{b}$ other than a part from b part, what do you think?
S : $\frac{a}{b}$ is equal to a per b with $a \neq 0$, and $b > a$

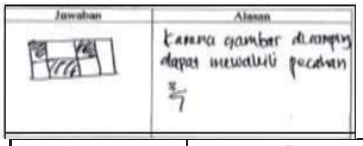
The excerpt of the interview shows that he has not been able to give meaning to fractions or those represented in the form of images even though the subject cannot provide clear and complete reasons so that the subject can formally conclude a form of generalization of the fraction concept even though it is not clear and complete. From the examples given in the interview, the subject can express problems related to the concept of fractions in various representations and use them in real life. It indicates that subject at the observing understanding layer. In this layer, the student can reflect and coordinate formal activities in the formalizing layer so that they are capable of using it on the problems faced and express it as the theorem (1).

Results of the Answers to the Subject of Low Communication Skills in Solving Fraction Problems

The answer result from the subject with low communication skills in solving fraction problems is shown in Fig. 4 below.



Answers	Reasons
$\frac{5}{9}$	Because the shaded area is a numerator and the unshaded area is a denominator So the fraction value is $\frac{5}{4}$



Answers	Reasons
	Because the picture on the left can represent the fraction $\frac{3}{7}$

FIGURE 4. The work result of subject with low communication skills

The subject of low communication skills in solving fraction problems cannot provide clear reasons. The subject understands the concept of fractional numbers which are numbers expressed as $\frac{a}{b}$ with a is the numerator and b is the

denominator. Although the reasons given are not clear, the subject is considered to be in the primitive knowing understanding layer. This layer describes the process of initial attempts to understand a new concept by bringing previous knowledge to the next understanding layer which involves (1), (15), (21). This subject could give the name or symbol to the fraction $\frac{5}{9}$ from the triangular image in the fraction problem related to the visual problem. Besides, the subject could also produce a rectangular image which represents the fraction $\frac{3}{7}$ in the fraction problem related to the verbal problem. It means that this subject at the image making an understanding layer. When the student work by involving images/diagrams through specific examples to develop their ideas, it means that the students is in the image making understanding layer (22). According to the reasons given, it shows that the subject does not have the idea of constructing mental images in solving fraction problems. When mentioning "numerator" and denominator" and ":to represent", it means that the subject can manipulate and combine the concept of fraction and division in the form of representations based on the nature of number operation. It indicates that subject is in the property noticing understanding layer. According to Pirie & Kieren (1), they state that students are in the property noticing understanding layer when they can manipulate and combine aspects of a topic to form properties that are relevant and specific to the concept.

Pay attention to the next interview excerpt conducted by the researchers on the subject of low communication skills in solving fraction problems.

P : Alright, if that so, from $b \neq 0$, and $b > a$, what can you conclude? If a, b are integers.

S : $\frac{a}{b} = a$ is divided by b with a is the numerator and b is the denominator, $b \neq 0$ and $b > a$

P : Can you make a real example from the fraction that you defined?

S : The example is when a cake is divided into 4 and it is given to 4 children. Then each child will receive $\frac{1}{4}$

P : Is the answer on number 1 actually $\frac{5}{4}$ or $\frac{5}{9}$? Why is your answer not similar from the answer to the reason?

S : $\frac{5}{9}$ Mam, I forgot to revise for the reason.

Based on the result of the interview excerpt above, it shows that subject can give meaning to fraction and the ones represented and is able to draw conclusions formally as a form of generalization of the fraction concept. This indicates that subject is in the formalizing understanding layer. In this layer, the students are ready and able to clearly state and appreciate a definition or formal mathematical algorithm (1). At the last part of the conversation, the subject can provide an example, so this indicates that the subject at the observing understanding layer. According to Pirie & Kieren (1), the students are in the observing understanding layer when the students can reflect and coordinate formal activities in the formalizing layer so that they are able to use them on the problems faced and express them as theorem. According to Sengul & Argat (23) in his research, he said that to improve students' understanding, it could be done by generalizing the images in their minds.

CONCLUSION

The result of this study which aims at exploring the seventh-grader students conceptual understanding layer based on their communication skills in solving fraction problems brings to a conclusion that Subjects with high communication skills have reached layer of understanding namely primitive knowing, image having, image making, property noticing, formalizing, observing, structuring and subject has passed the second "don't need" boundaries in solving fraction problems because is able to give meaning to each fraction as well as the ones represented and draw conclusions formally as a form of generalization from the fraction concept without having to show particular images specifically. Subject with moderate communication skills reached layer of understanding primitive knowing, image making, image having, property noticing, and observing. The subject of low communication skills reached layer of understanding primitive knowing, image making, property noticing, and formalizing.

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